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A NEW INITIAL BIAS DELETION RULE

by

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and

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TECHNICAL REPORT NO. 26

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A NEW INITIAL BIAS DELETION RULE

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ABSTRACT

A new initial bias deletion rule is described. Theoretical motivation for the rule is given, and some directions on further research are provided.

1. INTRODUCTION

Consider a real-valued stochastic process $Y = \{Y(t) : t \geq 0\}$ which is ergodic, in the sense that there exists a finite (deterministic) constant μ such that

$$\overline{Y}(t) \equiv \frac{1}{t}\int_0^t Y(s)ds \Rightarrow \mu \qquad (1.1)$$

as $t \to \infty$. The steady-state simulation problem is concerned with the estimation of μ, and production of associated confidence intervals.

The limit relation (1.1) immediately suggests using the estimator $\overline{Y}(t)$, namely the sample mean of Y up to time t. A difficulty with this approach is that

$$E\overline{Y}(t) \neq \mu \qquad (1.2)$$

in general. The bias in $\overline{Y}(t)$ is a consequence of using an atypical initial condition to start the simulation; atypical refers here to an initial condition which is not typical of the steady-state of the process. For example, queueing simulations of open queueing networks are typically initiated with all stations idle. In this setting, one generally sees the customer population build up to a "steady-state".

The bias difficulty (1.2) is often called the initial bias problem. This initial bias problem has historically attracted considerable attention within the simulation community.

One way to deal with this initial bias difficulty is to delete those initial observations most severely "contaminated" by the initial condition; such methods are called initial bias deletion rules. In Section 2, we briefly describe a new initial bias deletion rule, which we believe has some desirable theoretical properties. Section 3 offers some conclusions about the rule. An expanded version of the work described here will be available in a forthcoming paper (Glynn and Iglehart (1988)).

2. A NEW INITIAL BIAS DELETION RULE

Suppose that $Y = \{Y(t) : t \geq 0\}$ can be represented as $Y(t) = f(X(t))$, where $X = \{X(t) : t \geq 0\}$ is a continuous-time Markov chain on state space $S = \{0, 1, 2, \ldots\}$. If X is positive recurrent and irreducible, then

$$\mu = \sum_{i \in S} \pi_i f(i)$$

where the π_i's are the stationary probabilities of X. Suppose V is a r.v. independent of X such that

$$P\{V = i\} = \pi_i,$$

and let

$$T = \inf\{t > 0 : X(t) = V\}.$$

We claim that $\{X(T+t) : t \geq 0\}$ is a stationary continuous-time Markov chain, so that the post-T process is in "steady-state". To see this, observe that for suitably measurable functions $k(\,)$,

$$\begin{aligned} Ek(X(T+t) : t \geq 0) &= \sum_{i \in S} E\{k(X(T+t) : t \geq 0) | V = i\} \ \pi_i \\ &= \sum_{i \in S} \pi_i \ E\{k(X(T_i + t) : t \geq 0)\} \end{aligned}$$

where $T_i = \inf\{t > 0 : X(t) = i\}$. By the strong Markov property

$$E\{k(X(T_i + t) : t \geq 0)\} = E\{k(X(t) : t \geq 0) | X(0) = i\}$$

so

$$Ek(X(T+t): t \geq 0) = \sum_{i \in S} \pi_i E\{k(X(t): t \geq 0) | X(0) = i\}$$

But the right-hand side is the expected value of the functional $k(\cdot)$ integrated against the probability distribution of a stationary continuous-time Markov chain.

Thus, if one could generate such a T, the initial bias deletion problem would be completely solved by deleting all observations collected prior to time T. Of course, since T involves using the stationary probabilities explicitly, construction of such a random time T in a practical context is infeasible.

As a consequence, it makes sense not to use this rule explicitly, but instead to attempt to "mimic" it. Rather than using the π_i's to generate T, we use the empirical steady-state distribution's $\hat{\pi}_i$'s. Specifically, let

$$\hat{\pi}_i(t) = \frac{1}{t}\int_0^t I(X(s) = i) ds.$$

Let $V(t)$ be the r.v. with the conditional distribution

$$P\{V(t) = i | X\} = \hat{\pi}_i(t)$$

and let

$$T(t) = \inf\{s \geq 0 : X(s) = V(t)\}.$$

The idea is then to delete, from the data set $\{Y(s) : 0 \leq s \leq t\}$, all observations collected prior to $T(t)$. This leaves the simulator with the (approximately) "steady-state" time series $\{Y(s) : T(t) \leq s \leq t\}$.

3. CONCLUDING REMARKS

We expect that the deletion rule $T(t)$ described in Section 2 mimics the rule given by T, in the sense that

$$P\{(X(T(t)+u) : u \geq 0) \in \cdot\} \Rightarrow \sum_{i \in S} \pi_i P\{(X(u) : u \geq 0) \in \cdot\}$$

as $t \to \infty$. However, this does not necessarily imply that the point estimate

$$\tilde{Y}(t) = \frac{1}{t - T(t)}\int_{T(t)}^t Y(s) ds$$

obtained after deletion of the initial segment $[0, T(t)]$ has low bias. The problem is that $\tilde{Y}(t)$ is a ratio of two random variables. Such ratio estimators are notoriously subject to bias difficulties.

This suggests that further theoretical and empirical study is necessary, in order to determine the efficacy of the deletion estimator $\tilde{Y}(t)$. This work will be reported in Glynn and Iglehart (1988).

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